This year's theme:

Using Maths to Fight COVID-19

1. A recent study found that if we were able to collect all of the particles of the novel coronavirus, it would be able to fit into a coke can. According to the assumption that the average volume of a coronavirus is 10^{-15}cm^3, and that all of the coronaviruses in the world has a volume of 300ml, the number of coronavirus in the world can be written in the form $a \cdot 10^b$, where a is a one-digit integer and b is an integer. What is the value of $a + b$?

2. The fee of a particular school is \$40000 per student per year. In 2020, due to the pandemic, students had to miss 25% of the school year. To compensate, the school reduced the school fees by 20%. What percentage of the original revenue does the school make in 2020 if there were 2500 children in school?

3. Inside your drawers there are different coloured masks. There are 8 blue masks, 8 black masks, 6 white masks and 5 green masks. Assume that you take out a mask from your drawer without looking at it and you take out one mask per day without putting it back. How many days would it take to guarantee that you draw out the colour of a mask that you wore before already?

4. A Community Center is distributing free masks. There are 8 adult-sized masks and 5 child-sized masks in stock. Let a be the number of adults and c be the number of children collecting the masks. How many possible ordered pairs (a, c) are there such that each adult-child pair receives 3 masks?

5. On one particular day, there were in total 50 thousand COVID cases in city A and 10 thousand in City B. If the number of people infected in City A grows at a steady rate of 2% per day, and in City B 17% per day, how many days, rounded to the nearest integer, will it take for there to be an equal number of cases in both cities?

6. The population of an area of a city measuring 14 miles by 30 miles is vaccinated against COVID. The government wants to vaccinate the population of the 2-mile wide strip surrounding the vaccinated area within the next two weeks. It takes four shipments of the vaccine to cover 1 square mile of the city, and the shipments cost \$50 each. How many hundreds of dollars will it cost the government to vaccinate its target area?
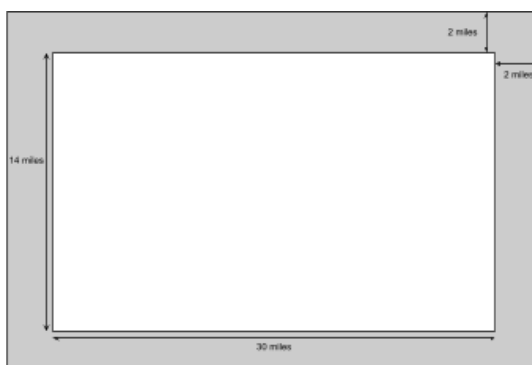
7. The sensitivity (accuracy of correctly identifying someone infected) and the specificity (accuracy of correctly identifying someone not infected) of a PCR or molecular test for COVID-19 are both around 99%. Singapore has a population of 5.7 million people, among which 57 thousand are infected with the virus. What is the probability for a person with a positive result to be actually infected? Give your answer as a percentage, without the percentage symbol.

8. The diameter of a SARS-CoV-2 particle can range from 50nm to 140nm. A cube-shaped container was built with a side length of 60nm such that a sphere-shaped SARS-CoV-2 particle can fit perfectly inside it, just touching each side of the container. The remaining space in the container is then filled with a special chemical, the volume of which can be represented in the form of $a \cdot 10^3 \cdot (b - \pi)$ nm^3, where a and b are positive integers. What is the value of a?

9. The spread of COVID-19 in a particular city follows a strange pattern. Let i_n denote the number of net cases (number of infected minus number of cured) on day n. It follows that $i_1 = 0$ and $i_{n+1} = i_n + n^2 - n - 1$ for all $n \geq 2$. What is the number of net cases after two weeks (i.e. on day 14)?

10. While the great mathematics teacher Julie Dale was researching ways to forecast the COVID-19 outbreak using polynomials, she discovered an interesting function, which was later named the "Dale function" after her. The Dale function has the property that $D(x + 4) = D(x - 4)$ for all real numbers x. If a Dale function $f(x)$ has 6 distinct real roots, what is the sum of these roots?