1. What is the value of x in the following equations?

$$\begin{cases} 20x + 20y = 2020 \\ 20x + 21y = 2021 \end{cases}$$

   (A) 1   **(B)** 2   **(C)** 99   **(D)** 100   **(E)** 101

2. What is the absolute difference between $2^3 - 3^2$ and $3^4 - 4^3$?

   (A) 15   **(B)** 16   **(C)** 17   **(D)** 18   **(E)** 19

3. What is the value of

$$\frac{2021!}{2020!} - \frac{2020!}{2019!} + \frac{2019!}{2018!} - \frac{2018!}{2017!} + \cdots + \frac{3!}{2!} - \frac{2!}{1!}?$$

   (A) 0   **(B)** 1   **(C)** 1010   **(D)** 1011   **(E)** 2021

4. If $a \oslash b = \sqrt{\frac{a}{b} + \frac{2a}{b}}$, what is the value of $12 \oslash 4$?

   (A) 3   **(B)** $\frac{6}{2\sqrt{2}}$ **(C)** 1   **(D)** $\sqrt{\frac{3}{2}}$ **(E)** 9

5. What is the value of

$$\frac{2021 \cdot 2019}{2020}?$$

   (A) $2019\frac{1}{2020}$ **(B)** $2019\frac{2019}{2020}$ **(C)** $2020\frac{1}{2020}$ **(D)** $2020\frac{2019}{2020}$ **(E)** $2021\frac{1}{2020}$

6. Lord Mountbatten was the president of the UWC movement from 1967 to 1978 and under his personal involvement, UWCSEA Dover was founded. He was also an Admiral for the British Navy in WW2 and was involved in Project Habakkuk, a project to create a giant aircraft carrier made out of ice mixed with wood pulp. The ice aircraft carrier was planned to be 3.2km long, but only had a speed of 6 knots. 1 knot will be approximated as 1.85km/h. If Mountbatten were to sail the carrier from Alexandra, Egypt to Colombo, Sri Lanka, which has a distance of 7480km, and then Colombo, Sri Lanka to Singapore, which has a distance of 3140km, which of the following is closest to the ratio of the time of the journey from Alexandra to Colombo to the time of the journey from Colombo to Singapore, assuming the carrier travelled at 6 knots throughout both journeys?

   (A) 0.3   **(B)** 0.4   **(C)** 0.5   **(D)** 2   **(E)** 2.4

7. The Euler's identity is considered to be one of the most beautiful formulas in mathematics. It states that:

$$e^{i\pi} + 1 = 0$$

   where i is the imaginary number which equals $\sqrt{-1}$, and e is the Euler's number which rounds to 2.718. What is the value of

$$e^{i\pi} + e^{2i\pi} + e^{3i\pi} + \cdots + e^{2019i\pi} + e^{2020i\pi} + e^{2021i\pi}?$$

   (A) -2021 **(B)** -1 **(C)** 0   **(D)** 1   **(E)** 2021

8. If p is a prime number, which of the following is never a prime number?

   (A) $p+1$ **(B)** $p+3$ **(C)** $p+5$ **(D)** $p+7$ **(E)** All of the above

9. Which of the following is equal to

$$\left(\frac{1}{a^2}\right) \cdot \left(\frac{1}{1 + \frac{1}{a}}\right) \cdot \left(\frac{1}{1 - \frac{1}{a}}\right)?$$

   (A) $\frac{1}{a^2}$ **(B)** $\frac{1}{a^2+1}$ **(C)** $\frac{1}{(a+1)(a-1)}$ **(D)** $\frac{1}{a(a-1)}$ **(E)** $\frac{1}{(a-1)^2}$

10. A rectangle has a length of 16cm and a width of 8cm. How many squares with half the perimeter as the rectangle are required to obtain the same area?

   (A) $\frac{30}{9}$ **(B)** $\frac{32}{9}$ **(C)** $\frac{11}{3}$ **(D)** $\frac{34}{9}$ **(E)** 4

11. The circumference of a circle passes through the vertices of a rectangle with a length of 24 and a width of 10. What is the radius of this circle?

   (A) 12   **(B)** 13   **(C)** 22   **(D)** 24   **(E)** 26

12. How many values from 1 to 6 inclusive could a take such that the equation $y = x^2 + ax + 4$ does not touch or cross the x axis?

   (A) 2   **(B)** 3   **(C)** 4   **(D)** 5   **(E)** 6

13. There are 80 students studying mathematics in UWCSEA Dover and all of them have to study either trigonometry or calculus or both at the same time. 35 students study trigonometry and 20 students do not study calculus. How many students are studying both at the same time?

   (A) 15   **(B)** 25   **(C)** 45   **(D)** 55   **(E)** 60

14. There are five positive integers written on a whiteboard in ascending order. Both the median and mode are 8. The mean of the four smallest numbers is 10 and the mean of the four largest numbers is 20. What is the greatest possible value of the largest number?

   (A) 30   **(B)** 40   **(C)** 41   **(D)** 47   **(E)** 48

15. In the United World College of Super Eccentric Adults, math teachers always lie on Mondays, Tuesdays and Wednesdays, while English teachers always lie on Thursdays, Fridays and Saturdays. When I asked what day it was, a math teacher said: "I lied yesterday." An English teacher also said: "I lied yesterday." What day was it?

   (A) Tuesday   **(B)** Wednesday   **(C)** Thursday   **(D)** Friday   **(E)** Saturday

16. Amilia is doing a math trick with her friend Becka. She first asks Becka to think of a number. Becka then multiplies her number by 3, adds 108, divides the result by 2, and multiply the result by 12. After this, Amilia asks Becka to divide the result by m, then add n to this result. The final number turned out to be 150 more than 2 times the number Becka chose originally. What is $m + n$?

   (A) 87   **(B)** 92   **(C)** 117   **(D)** 121   **(E)** 123

17. A function $f(x)$ is defined recursively by $f(1) = 2$ and

$$f(x) = \frac{f(x-1) - 1}{f(x-1) + 1}$$

   for all $x \geq 2$. What is the value of $f(2021)$?

   (A) -3 **(B)** $-\frac{1}{2}$ **(C)** $\frac{1}{3}$ **(D)** 2   **(E)** 3

18. Mr. Sparks gave Amy and Ben 2021 real numbers, conveniently numbered i_1 to i_{2021}. Amy calculated the product of $(i_1 + i_2 + \cdots + i_{2020}) \cdot (i_2 + i_3 + \cdots + i_{2021})$ and called her number A. Ben calculated the product of $(i_1 + i_2 + \cdots + i_{2021}) \cdot (i_2 + i_3 + \cdots + i_{2020})$ and called his number B. If $i_1 = -20$ and $i_{2021} = 21$, what is the value of $|A - B|$?

   (A) 20   **(B)** 41   **(C)** 420   **(D)** 2021   **(E)** More information required

19. Singaporean vehicle registration plates start off with the letter 'S'. It is followed by two alphabet letters except for 'I' and 'O', as these could be confused with the numbers '1' and '0'. After this, it is followed by four numbers with the condition that the leading number can not be a '0'. The final letter is a checksum letter, which could be any alphabet except for 'F', 'I', 'N', 'O', 'Q', 'V' and 'W'. How many possibilities are there for a typical vehicle registration plate in Singapore which contains only 1 letter 'N', only 1 letter 'A', at least 1 '1' and at least 2 '2's?

   (A) 4420   **(B)** 6760   **(C)** 8892   **(D)** 9120   **(E)** 9360

20. If

$$\frac{a}{a^2 + a + 1} = \frac{1}{2021}$$

   for some real number a, what is the value of

$$\frac{a^2}{a^4 + a^2 + 1}?$$

   (A) $\frac{1}{2020}$ **(B)** $\frac{1}{2021^2}$ **(C)** $\frac{1}{2020^2+1}$ **(D)** $\frac{1}{2020^2}$ **(E)** $\frac{1}{2020^2-1}$

21. For a positive integer n, the expression

$$\frac{2 + 5 + 8 + \cdots + (3n - 1)}{4 + 7 + 10 + \cdots + (3n + 1)} = \frac{2017}{2021}$$

   What is the value of n?

   (A) 668   **(B)** 669   **(C)** 670   **(D)** 671   **(E)** 672

22. There are three points on the graph of $x^2 - 8x + 12$ with an x value of a, b and c, which are distinct integers. The median y value of the three points is 21, and the x value of that point is also the median x value of the three points. The slope of the line connecting the point with the greatest y value and the point with the least y value is -4. What is the value of $a + b + c$?

    (A) 3   **(B)** 5   **(C)** 7   **(D)** 12   **(E)** 15

23. In a cube, let line a go through one vertex of the cube to the opposite vertex. Let line b start in the midpoint of one of the edges with a vertex which line a goes through and goes through the midpoint of the opposite edge. Let line c go through the other vertex on one of the edges with the vertex which line a goes through and goes through the vertex located on the opposite side of this vertex. Let α be the angle less than or equal to $90°$ between line a and line b at their intersection, and let β be the angle less than or equal to $90°$ between line a and line c. What is the value of $\cos\alpha + \cos\beta$?

    (A) 0   **(B)** $\frac{1+\sqrt{3}}{3}$ **(C)** $\frac{1+\sqrt{6}}{3}$ **(D)** $\frac{2+\sqrt{6}}{3}$ **(E)** 1

24. Square $ABCD$ has a side length of 1. E, F, G, H are points on line segments AB, BC, CD and DA, respectively, such that AE, BF, CG and DH are equal. If the area of square $PQRS$ formed by line segments AG, BH, CE and DF is $\frac{1}{13}$ the area of $ABCD$, what is the length of AE?

Not drawn to scale
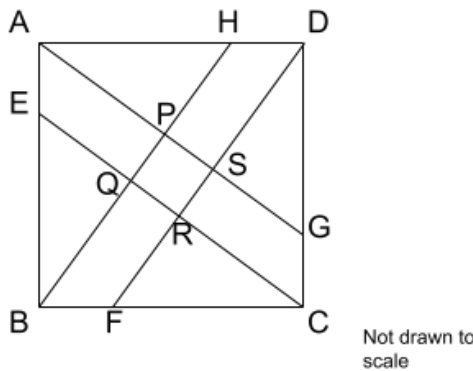
    (A) $\frac{1}{3}$ **(B)** $\frac{1}{\sqrt{13}}$ **(C)** $\frac{\sqrt{3}}{6}$ **(D)** $\frac{1}{5}$ **(E)** $\frac{1}{4}$

25. What is the total number of acute triangles (all 3 angles are less than $90°$) obtained by connecting any 3 vertices of a pentadecagon (15-gon)?

    (A) 28   **(B)** 140   **(C)** 420   **(D)** 455   **(E)** 2730